เนื้อหาการสอบและตัวอย่างข้อสอบแข่งขันชิงทุน Certificate in Quantitative Finance (“CQF”) ครั้งที่ 2

<u>เนื้อหาการสอบคณิตศาสตร์</u>

1. Calculus

    - Function
    - Limit
    - Differentiation
    - Taylor series
    - Integration
    - Complex numbers
    - Multivariable calculus

    References

    https://ocw.mit.edu/ans7870/resources/Strang/Edited/Calculus/Calculus.pdf

    https://www.sac.edu/FacultyStaff/HomePages/MajidKashi/PDF/MATH_150/Bus_Calculus.pdf

2. Linear algebra
    - Vectors
    - Matrices
    - Inverse
    - Determinant
    - Linear dependence
    - Solution of linear system
    - Orthogonality
    - Eigenvalues and eigenvectors

    References

    https://www.math.ucdavis.edu/~linear/linear-guest.pdf

    https://web.stanford.edu/~boyd/vmls/vmls.pdf

3. Differential equations

   - Ordinary differential equation (ODE)
   - Partial differential equation (PDE)
   - Initial and boundary value problems
   - Solution of ODE
   - Solution of PDE

   Ordinary differential equations references

   https://ncert.nic.in/ncerts/l/lemh203.pdf

   http://www.math.toronto.edu/~selick/B44.pdf

   https://www.math.hkust.edu.hk/~machas/differential-equations.pdf

   Partial differential equations references

   https://www.math.uni-leipzig.de/~miersemann/pdebook.pdf

4. Probability and statistics

   - Axioms and basic rules
   - Random variables
   - Expected values, variances, covariance, and correlation
   - Cumulative distribution function (CDF)
   - Probability density function (PDF)
   - Poisson distribution
   - Normal and lognormal distributions
   - Central limit theorem
   - Maximum likelihood
   - Regression
   - Random processes

References

http://users.encs.concordia.ca/~doedel/courses/comp-233/slides.pdf

https://cis.temple.edu/~latecki/Courses/CIS2033-Spring13/Modern_intro_probability_statistics_Dekking05.pdf

http://www.ru.ac.bd/stat/wp-content/uploads/sites/25/2019/03/501_06_Rohatgi_An-Introduction-to-Probability-and-Statistics-Wiley-2015.pdf

ตัวอย่างข้อสอบคณิตศาสตร์

1. Three values of x and y are to be fitted in a straight line in form $y = a + bx$ by the method of least squares. Given, $\sum x = 6, \sum y = 21, \sum x^2 = 14$, and $\sum xy = 46$, the values of a and b respectively are

   (a) 2 and 3 (b) 1 and 2

   (c) 2 and 1 (d) 3 and 2

2. The solution of the partial differential equation $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ is of the form

   (a) $C\cos(kt)\left[C_1 e^{\sqrt{k/\alpha}x} + C_2 e^{-\sqrt{k/\alpha}x}\right]$

   (b) $C\,e^{kt}\left[C_1 e^{\sqrt{k/\alpha}x} + C_2 e^{-\sqrt{k/\alpha}x}\right]$

   (c) $C\,e^{kt}\left[C_1\cos\left(\sqrt{k/\alpha}\right)x + C_2\sin\left(-\sqrt{k/\alpha}\right)x\right]$

   (d) $C\sin kt\left[C_1\cos\left(\sqrt{k/\alpha}\right)x + C_2\sin\left(-\sqrt{k/\alpha}\right)x\right]$

3. i^i, where $i = \sqrt{-1}$, is given by

   (a) $e^{\frac{\pi}{2}}$ (b) $e^{-\frac{\pi}{2}}$

   (c) $\frac{\pi}{2}$ (d) $e^{2\pi}$

4. The value of the integral $\int_0^{2\pi} \frac{3}{9+\sin^2\theta}\,d\theta$ is

   (a) 2π (b) $2\sqrt{10}\pi$

   (c) $\sqrt{10}\pi$ (d) $\frac{2\pi}{\sqrt{10}}$

5. The sum of the infinite series, $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ is

(a) π (b) ∞

(c) 4 (d) $\frac{\pi^2}{4}$

6. The quadratic approximation of $f(x)=x^3-3x^2-5$ at the point $x=0$ is

(a) $-3x^2-5$ (b) $3x^2-6x-5$

(c) -$3x^2+6x-5$ (d) $3x^2-5$

7. The general solution of the differential equation $\frac{d^4y}{dx^4}-2\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$

(a) $y=(c_1-c_2x)e^x+c_3\cos x+c_4\sin x$

(b) $y=(c_1+c_2x)e^x-c_3\cos x+c_4\sin x$

(c)) $y=(c_1+c_2x)e^x+c_3\cos x+c_4\sin x$

(d)) $y=(c_1+c_2x)e^x+c_3\cos x-c_4\sin x$

8. The solution of the differential equation $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=9x+6$ with C_1 and C_2 as constant is

(a) $y=(C_1x+C_2)e^{-3x}$ (b) $y=C_1e^{3x}+C_2e^{-3x}$

(c) $y=(C_1x+C_2)e^{-3x}+x$ (d) $y=(C_1x+C_2)e^{3x}+x$

9. The probability density function of a random variable X is

$$f_X(x)=\begin{cases}\frac{x}{4}(4-x)^2, & 0\le x\le 2\\ 0, & otherwise.\end{cases}$$

The expected value of the random variable is

(a) 16/15 (b) 15/16

(c) 4/15 (d) 5/16

10. A normal random variable X has the following probability density function

$$f_X(x) = \frac{1}{\sqrt{8\pi}} \exp\left(-\frac{(x-1)^2}{8}\right), -\infty < x < \infty,$$

Then $\int_1^{\infty} f_X(x)dx = ?$

(a) 0 (b) 1/2

(c) $1 - \frac{1}{e}$ (d) 1

11. The function $f(x,y) = x^2y - 3xy + 2y + x$ has

(a) No local extremum

(b) One local maximum but no local minimum

(c) One local minimum but no local maximum

(d) One local minimum and one local maximum

12. Find $\lim_{x\to\infty} \left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)$

(a) 0 (b) 1

(c) 2 (d) ∞

13. The eigenvalues of the matrix M given below are 15, 3 and 0. $M = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, the value of the determinant of the matrix is

(a) 20 (b) 10

(c) 0 (d) -10

14. The inverse of the matrix $S = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & -2 \\ 0 & 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

15. Let $f(x,y) = \frac{ax^2+by^2}{xy}$, where a and b are constants, If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ at $x = 1$ and $y = 2$, then the relation between a and b is

(a) $a = b/4$ (b) $a = b/2$

(c) $a = 2b$ (d) $a = 4b$

<u>เฉลยตัวอย่างข้อสอบคณิตศาสตร์</u>

1. d	2. b	3. b	4. d	5. b	6. a	7. c	8. c
9. a	10. b	11. a	12. d	13. c	14. d	15. d	

เนื้อหาการสอบ Python Programming

- Expression, variables, and sequential programming
- Subroutine
- Selections
- Repetition
- List
- File input and text processing
- NumPy
- Introduction to OOP

ตัวอย่างข้อสอบ Python Programming

1. Which symbol is used in Python to add comment?
    a. //
    b. $
    c. /*...*/
    d. #
    e. !
2. What is the output of the following program?

```
1: def func(x):
2:   if x % 2 == 0:
3:     return 0
4:
5: print(func(1) + func(2))
```

    a. True
    b. 0
    c. 2
    d. 3
    e. None of the above choices is correct

3. What is the output of the following program?

```
1: x = 0
2: while x < 3:
3:   print(x)
4:   x += 1
5: else:
6:   print(0)
```

   a. False
   b. 0 1 2 3 0
   c. 0 1 2 0
   d. 0 1 2 3
   e. 0 1 2

4. Which '+' sign has a different meaning from the others?
   a. 1 + 2
   b. 2.0 + 4.0
   c. int('3') + int('4')
   d. '4' + '8'
   e. float('5') + float('10')

5. Which choice is a valid statement?
   a. x = '4' + int('5')
   b. x = '4' - int('5')
   c. x = '4' * int('5')
   d. x = '4' / int('5')
   e. x = '4' ** int('5')

6. What should be filled in (1) to make the output of the following program be “True”?

```
1: a = [1]
2: b = a
3: a[0] = 0
4: print(____(1)____)
```

a. len(a) == len (b) - 1

b. b [0] + 1 ==a [0]

c. a [0] + 1 ==b [0]

d. a[0] == b[0]

e. a[0] == b[1]

7. What is the output of the following program?

```
1: a = 3
2: if a > 5:
3:   print('A')
4:   if a == 3:
5:     print('B')
6:   else:
7:     print('C')
```

a. A

b. B

c. C

d. A
   B

e. None of the above choices is correct

8. From the following functions, what is the equivalence of **f(g(y))**?

```
1: def f(x):
2:   return 2*x+3
3: def g(x)
4:   return 10*x
```

a. $20y^2 + 3$

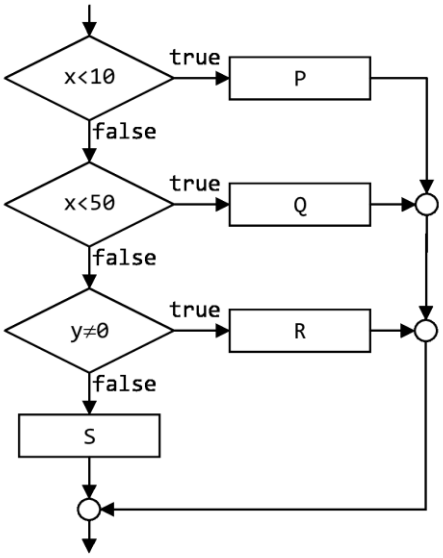
b. $20y + 3$

c. $20y^2 + 30$

d. $20y + 30$

e. $10y^2 + 3$

9. From the following flowchart, which choice gives the best conditions of x and y when the block S is executed?



a. y is not 0

b. y equals 0

c. x is less than 50 and y is not 0

d. x is greater than or equals to 50 and y equals 0

e. y is not equal to x

10. Which choice correctly calculates the expression:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}$$

a.
```
total = 0
for x in [1, 2, 3]:
  total = total + 1/x
print(total)
```

b.
```
total = 0
for x in range(3):
  total = total + 1/x
print(total)
```

c.
```
a = [1,2,3]
for x in a:
  x = 1 / x
  total = sum(a)
print(total)
```

d.
```
a = [1,2,3]
a = 1/a
total = sum(a)
print(total)
```

e.
```
a = [1,2,3]
```

```
for x in range(3):
  a[x] = a[x]/x
  total = sum(a)
print(total)
```

11. What is the output of the following program?

```
1: x = 4.5
2: y = 2
3: print(x//y)
```

a. 2.0
b. 2.25
c. 9.0
d. 20.25
e. 21

12. Consider the following 4 statements, which choice is the correct order of a program that calculates area of a square?

```
1: print(f'area is {area}')
2: l = float(input(msg))
3: area = l * l
4: msg = 'Input side length: '
```

a. 1, 2, 3, 4
b. 2, 3, 4, 1
c. 2, 4, 3, 1
d. 4, 2, 3, 1
e. 4, 3, 2, 1

13. From the program fragment below, which choice is correct?

```
1: class A:
2:   def __init__(self, i=100):
3:     self.i=i
4: class B(A):
5:   def __init__(self,j=0):
6:     self.j=j
7:
8: b = B()
9: print(b.i)
```

a. Class B inherits all the data fields of class A.

b. Class B needs an Argument.

c. Object 'b' automatically has data fields 'I' and 'j'.

d. The data field 'j' cannot be accessed by object 'b'.

e. Class B is inheriting class A but the data field 'I' in A cannot be inherited

14. What is the output of the following program?

```
1: import numpy as np
2: a = np.array([[1,2],[3,4]])
3: b = np.array([[5,6],[7,8]])
4: print(a*b)
```

a.
```
[[1 2 5 6]
 [3 4 7 8]]
```

b.
```
[[1 2]
 [3 4]
```

```
[5 6]

[7 8]]
```

c.
```
[[5 12]

 [21 32]]
```

d.
```
[[19 22]

 [43 50]]
```

e. None of the above choices is correct

15. Which choice is the contents of the file **data.txt** that produces the result as in the following Python shell session?

```
1: >>> import numpy as np
2: >>> data = np.loadtxt("data.txt", delimiter=",")
3: >>> data.size
4: 6
5: >>> data.shape
6: (2, 3)
7: >>> data[1]
8: array([ 56., 27., 61.])
9: >>> data.T[1]
10: array([ 17., 27.])
```

a.

```
17,17,79

56,27,61

32,26,88
```

b.

```
56,27,61

17,27,90
```

c.

```
56,17

27,27

61,90
```

d.

```
34,56

17,27

83,61
```

e.

```
34,17,83

56,27,61
```

เฉลยตัวอย่างข้อสอบ Python Programming

1. d	2. e	3. c	4. d	5. c	6. d	7. e	8. b
9. d	10. a	11. a	12. d	13. e	14. c	15. b	